

# Photonic portal to hidden sector and a parity-preserving option

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## Abstract

In the case of previously proposed idea of photonic portal to hidden sector, the parity in this sector may be violated. We discuss here two new options within our model, where the parity is preserved. The first of them is not satisfactory, as not displaying a full relativistic covariance. The second seems to be satisfactory.

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## 1. Introduction

In previous papers [1,2], we have proposed a model of hidden sector of the Universe, consisting of sterile spin-1/2 Dirac fermions ("sterinos"), sterile spin-0 bosons ("sterons"), and sterile nongauge mediating bosons ("A bosons") described by an antisymmetric-tensor field (of dimension one) weakly coupled to steron-photon pairs and, more obviously, to the antisterino-sterino pairs,

$$-\frac{1}{2}\sqrt{f} \left( \varphi F_{\mu\nu} + \zeta \bar{\psi} \sigma_{\mu\nu} \psi \right) A^{\mu\nu}, \quad (1)$$

where  $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$  is the Standard-Model electromagnetic field (of dimension two), while  $\sqrt{f}$  and  $\sqrt{f} \zeta$  denote two dimensionless small coupling constants. Here, it is presumed that  $\varphi = \langle \varphi \rangle_{\text{vac}} + \varphi_{\text{ph}}$  with a spontaneously nonzero vacuum expectation value  $\langle \varphi \rangle_{\text{vac}} \neq 0$ . Such a coupling of photons to the hidden sector has been called "photonic portal" (to hidden sector). It provides a weak coupling between the hidden and Standard-Model sectors of the Universe. The photonic portal is an alternative to the popular "Higgs portal" (to hidden sector) [3].

The new interaction Lagrangian (1), together with the A-boson kinematic and Standard-Model electromagnetic Lagrangians, leads to the following field equations for  $F_{\mu\nu}$  and  $A_{\mu\nu}$  :

$$\partial^\nu \left[ F_{\mu\nu} + \sqrt{f} (\langle \varphi \rangle_{\text{vac}} + \varphi_{\text{ph}}) A_{\mu\nu} \right] = -j_\mu, \quad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \quad (2)$$

and

$$(\square - M^2) A_{\mu\nu} = -\sqrt{f} [(\langle \varphi \rangle_{\text{vac}} + \varphi_{\text{ph}}) F_{\mu\nu} + \zeta \bar{\psi} \sigma_{\mu\nu} \psi], \quad (3)$$

where  $j_\mu$  denotes the Standard-Model electric current and M stands for a mass scale of A bosons, expected typically to be large.

The field equations (2), called "supplemented Maxwell's equations", are modified due to the presence of hidden sector. This modification has a magnetic character, because the hidden-sector contribution to the total electric source-current  $j_\mu + \partial^\nu [\sqrt{f} (\langle \varphi \rangle_{\text{vac}} + \varphi_{\text{ph}}) A_{\mu\nu}]$  for the electromagnetic field  $A_\mu$  is a four-divergence giving no contribution to the total electric charge  $\int d^3x \{j_0 + \partial^k [\sqrt{f} (\langle \varphi \rangle_{\text{vac}} + \varphi_{\text{ph}}) A_{0k}]\} = \int d^3x j_0 = Q$ . In

particular, it can be seen that the vacuum expectation value  $\langle \varphi \rangle_{\text{vac}} \neq 0$  generates spontaneously a small sterino magnetic moment

$$\mu_\psi = \frac{f\zeta}{2M^2} \langle \varphi \rangle_{\text{vac}}, \quad (4)$$

though sterinos are electrically neutral. This is a consequence of an effective sterino magnetic interaction

$$- \mu_\psi \bar{\psi} \sigma_{\mu\nu} \psi F^{\mu\nu} \quad (5)$$

appearing, when the low-momentum-transfer approximation

$$A_{\mu\nu} \simeq \frac{\sqrt{f}\zeta}{M^2} \bar{\psi} \sigma_{\mu\nu} \psi \quad (6)$$

effectively implied by Eq. (3) is used in the interaction (1) with  $\varphi = \langle \varphi \rangle_{\text{vac}} + \varphi_{\text{ph}}$ .

## 2. Option of independent field components for $A$ bosons

In analogy with the familiar splitting of  $F_{\mu\nu}$  into  $\vec{E}$  and  $\vec{B}$ , we can split the field  $A_{\mu\nu}$  into the three-dimensional vector and axial fields  $\vec{A}^{(E)}$  and  $\vec{A}^{(B)}$  of spin 1 and parity  $-$  and  $+$ , respectively. Then,

$$(A_{\mu\nu}) = \begin{pmatrix} 0 & A_1^{(E)} & A_2^{(E)} & A_3^{(E)} \\ -A_1^{(E)} & 0 & -A_3^{(B)} & A_2^{(B)} \\ -A_2^{(E)} & A_3^{(B)} & 0 & -A_1^{(B)} \\ -A_3^{(E)} & -A_2^{(B)} & A_1^{(B)} & 0 \end{pmatrix}. \quad (7)$$

Similarly, for the spin tensor  $\sigma^{\mu\nu} = (i/2)[\gamma^\mu, \gamma^\nu]$  with  $\vec{\alpha} = (\alpha_k) = (\gamma^0 \gamma^k) = (i\sigma^{k0})$  and  $\vec{\sigma} = (\sigma_k) = \gamma_5 \vec{\alpha} = (1/2)(\varepsilon_{klm} \sigma^{lm})$  ( $k = 1, 2, 3$ ), we get

$$(\sigma^{\mu\nu}) = \begin{pmatrix} 0 & i\alpha_1 & i\alpha_2 & i\alpha_3 \\ -i\alpha_1 & 0 & \sigma_3 & -\sigma_2 \\ -i\alpha_2 & -\sigma_3 & 0 & \sigma_1 \\ -i\alpha_3 & \sigma_2 & -\sigma_1 & 0 \end{pmatrix}. \quad (8)$$

Then, the interaction (1) can be rewritten in the form

$$\left( \varphi \vec{E} - i\zeta \bar{\psi} \vec{\alpha} \psi \right) \cdot \vec{A}^{(E)} - \left( \varphi \vec{B} - \zeta \bar{\psi} \vec{\sigma} \psi \right) \cdot \vec{A}^{(B)}, \quad (9)$$

where  $\varphi = \langle \varphi \rangle_{\text{vac}} + \varphi_{\text{ph}}$  with  $\langle \varphi \rangle_{\text{vac}} \neq 0$ . Consequently, the first and second of supplemented Maxwell's equations (2) for photons can be split as follows:

$$\begin{aligned} \vec{\partial} \times (\vec{B} + \sqrt{f} \varphi \vec{A}^{(B)}) &= \partial_0 (\vec{E} + \sqrt{f} \varphi \vec{A}^{(E)}) + \vec{j} \quad , \quad \vec{\partial} \cdot (\vec{E} + \sqrt{f} \varphi \vec{A}^{(E)}) = j_0 \quad , \\ \vec{\partial} \times \vec{E} &= -\partial_0 \vec{B} \quad , \quad \vec{\partial} \cdot \vec{B} = 0 \end{aligned} \quad (10)$$

and the field equation (3) for  $A$  bosons as:

$$\begin{aligned} (\square - M^2) \vec{A}^{(E)} &= -\sqrt{f} (\varphi \vec{E} - i \zeta \bar{\psi} \vec{\alpha} \psi) \quad , \\ (\square - M^2) \vec{A}^{(B)} &= -\sqrt{f} (\varphi \vec{B} - \zeta \bar{\psi} \vec{\sigma} \psi) \quad , \end{aligned} \quad (11)$$

where  $\varphi = \langle \varphi \rangle_{\text{vac}} + \varphi_{\text{ph}}$  with  $\langle \varphi \rangle_{\text{vac}} \neq 0$ . Here,  $(j_\mu) = (j_0, -\vec{j})$  is the Standard-Model current ( $\vec{E} = -\partial_0 \vec{A} - \vec{\partial} A_0$  and  $\vec{B} = \vec{\partial} \times \vec{A}$  with  $(\partial_\mu) = (\partial_0, \vec{\partial})$  and  $(A_\mu) = (A_0, -\vec{A})$ ). Note that the source-free Eqs. (10) are, of course, the ordinary source-free Maxwell's equations.

The sterile  $A$  bosons described by the fields  $\vec{A}^{(E)}$  and  $\vec{A}^{(B)}$ , when they propagate freely in space ( $\sqrt{f} \rightarrow 0$ ), get the one-particle wave functions

$$\vec{A}_{\vec{k}_A}^{(E,B)}(x) = \frac{1}{(2\pi)^{3/2}} \frac{1}{\sqrt{2\omega_A}} \vec{e}^{(E,B)} e^{-ik_A \cdot x} \quad , \quad (12)$$

where  $k_A = (\omega_A, \vec{k}_A)$  with  $\omega_A = \sqrt{\vec{k}_A^2 + M^2}$ , while  $\vec{e}^{(E,B)}$  are linear polarizations of  $A^{(E)}$  and  $A^{(B)}$  bosons [2].

If the fields  $\vec{A}^{(E)}$  and  $\vec{A}^{(B)}$  are independent (as can be in Eqs. (11)), then these polarizations form two triples of orthonormal versors,

$$\vec{e}_a^{(E,B)} \cdot \vec{e}_b^{(E,B)} = \delta_{ab} \quad (a, b = 1, 2, 3) \quad , \quad \sum_{a=1}^3 e_{ak}^{(E,B)} e_{al}^{(E,B)} = \delta_{kl} \quad (k, l = 1, 2, 3) \quad (13)$$

with  $\vec{e}_a^{(E,B)} = (e_{ak}^{(E,B)})$  ( $a = 1, 2, 3$ ,  $k = 1, 2, 3$ ) [2]. If the parity is preserved by the new weak interaction (1) or (9) in hidden sector, the polarizations  $\vec{e}_a^{(B)}$  ought to be axial vectors, while  $\vec{e}_a^{(E)}$  are polar vectors. In this case, the axial and polar vectors are  $\vec{e}^{(B)} = (-e_{23}, -e_{31}, -e_{12})$  and  $\vec{e}^{(E)} = (-e_{10}, -e_{20}, -e_{30})$ , respectively, where  $e_{\mu\nu}$  ( $\mu, \nu = 0, 1, 2, 3$ ) describe the antisymmetric polarization tensors appearing in the  $A$ -boson relativistic free

wave function  $A_{\mu\nu\vec{k}_A}(x)$  split into  $\vec{A}_{\vec{k}_A}^{(E,B)}(x)$  given in Eqs. (12) (of course, there is a triplet of antisymmetric polarization tensors  $e_{\mu\nu a}$  ( $a = 1, 2, 3$ ) split into two triplets  $\vec{e}_a^{(E,B)}$  ( $a = 1, 2, 3$ )).

The axial  $\vec{e}_a^{(B)}$ , though defined carefully, are not practically realized for independent  $\vec{A}^{(E)}$  and  $\vec{A}^{(B)}$  fields. Therefore, in a real case, the field  $\vec{A}^{(B)}$  may play a role of an effective polar vector of parity  $-$  (like the field  $\vec{A}^{(E)}$ ) and so, *the parity may be maximally violated* by the second term of coupling (9) in hidden sector [2]. This violation appears formally, when  $\vec{e}_a^{(B)}$  are put polar (in spite of their original axial definition).

To be able to resign from such an option of independent field components for  $A$  bosons, some Maxwell-type relations between the massive fields  $\vec{A}^{(E)}$  and  $\vec{A}^{(B)}$  (of dimension one) may be tentatively discussed as a new option, but it turns out to be not satisfactory (Section 3). Some relations of equivalence between the fields  $\vec{A}^{(E)}$  and  $\vec{A}^{(B)}$  may define still a different option, satisfactory this time (Section 4).

### 3. Option of Maxwell-type relations between field components for massive $A$ bosons

Consider two three-dimensional fields  $\vec{X}^{(E)}$  and  $\vec{X}^{(B)}$  of spin 1 and parity  $-$  and  $+$ , respectively, satisfying the following set of first-order differential equations:

$$\begin{aligned}\vec{\partial} \times \vec{X}^{(B)} &= (\partial_0 + iM) \vec{X}^{(E)} + \vec{\rho} \ , \quad \vec{\partial} \cdot \vec{X}^{(E)} = \rho_0 \ , \\ \vec{\partial} \times \vec{X}^{(E)} &= (-\partial_0 + iM) \vec{X}^{(B)} \quad , \quad \vec{\partial} \cdot \vec{X}^{(B)} = 0 \ ,\end{aligned}\tag{14}$$

where  $(\rho_\mu) = (\rho_0, -\vec{\rho})$  is a four-vector fulfilling necessarily the condition

$$\vec{\partial} \cdot \vec{\rho} + (\partial_0 + iM)\rho_0 = 0\tag{15}$$

that would have the form of continuity equation if  $M$  were zero (the **div** operator is  $\vec{\partial} \cdot$ ). Then, acting on the first and third Eq. (14) by the **curl** operator  $\vec{\partial} \times$  and applying the identity

$$\vec{\partial} \times (\vec{\partial} \times \vec{X}) = \vec{\partial}(\vec{\partial} \cdot \vec{X}) - \Delta \vec{X}\tag{16}$$

( $\Delta \equiv \vec{\partial}^2$ ), we conclude after combining both equations that

$$\begin{aligned}
(\square - M^2)\vec{X}^{(E)} &= (\partial_0 - iM)\vec{\rho} + \vec{\partial}\rho_0 \equiv -\vec{J}^{(E)}, \\
(\square - M^2)\vec{X}^{(B)} &= -\vec{\partial} \times \vec{\rho} \equiv -\vec{J}^{(B)}
\end{aligned} \tag{17}$$

( $\square \equiv \Delta - \partial_0^2$ ). We can see that any solution to Eqs. (14) satisfies also Eqs. (17) (but not necessarily *vice versa*), so the former are a sufficient condition for the latter.

Now, it is inferred from Eqs. (11) and (17) that, if the identities

$$\begin{aligned}
(-\partial_0 + iM)\vec{\rho} - \vec{\partial}\rho_0 &\equiv \vec{J}^{(E)} \equiv \sqrt{f} \left( \varphi \vec{E} - i\zeta \bar{\psi} \vec{\alpha} \psi \right), \\
\vec{\partial} \times \vec{\rho} &\equiv \vec{J}^{(B)} \equiv \sqrt{f} \left( \varphi \vec{B} - \zeta \bar{\psi} \vec{\sigma} \psi \right)
\end{aligned} \tag{18}$$

were fulfilled, then our fields  $\vec{A}^{(E)}$  and  $\vec{A}^{(B)}$  might be used in place of  $\vec{X}^{(E)}$  and  $\vec{X}^{(B)}$  in Eqs. (14) and (17), where the former equations would be sufficient for the latter to hold (the latter would become Eqs. (11), being relativistic, as equivalent to the field equation (3) for  $A_{\mu\nu}$ ). Then,  $\vec{X}^{(E,B)} \equiv \vec{A}^{(E,B)}$  would have dimension one, while  $\vec{\rho}$  and  $\rho_0$  — dimension two (and  $\vec{J}^{(E,B)}$  — dimension three). In this case, however, the lhs of identities (18) (together with Eq. (15)) would imply new relations

$$\begin{aligned}
\vec{\partial} \times \vec{J}^{(B)} &= (\partial_0 + iM)\vec{J}^{(E)} - (\square - M^2)\vec{\rho}, \quad \vec{\partial} \cdot \vec{J}^{(E)} = -(\square - M^2)\rho_0, \\
\vec{\partial} \times \vec{J}^{(E)} &= (-\partial_0 + iM)\vec{J}^{(B)}, \quad \vec{\partial} \cdot \vec{J}^{(B)} = 0
\end{aligned} \tag{19}$$

which would be wrongly imposed by the rhs of Eqs. (18) on the independent fields  $\vec{E}$ ,  $\vec{B}$  and  $\varphi$ ,  $\psi$ ,  $\bar{\psi}$  (appearing then in  $\vec{J}^{(E)}$  and  $\vec{J}^{(B)}$ ). This is so, since they should be related only through dynamical relationships provided by the field equations (following from the total Lagrangian).

In order to avoid these unwanted nondynamical relations, one may impose Eqs. (14) — asymptotically ( $\sqrt{f} \rightarrow 0$ ) and softly — on the free wave functions (12) of massive  $A$  bosons, writing

$$\begin{aligned}
\vec{\partial} \times \vec{A}_{\vec{k}_A}^{(B)} &= (\partial_0 + iM)\vec{A}_{\vec{k}_A}^{(E)}, \quad \vec{\partial} \cdot \vec{A}_{\vec{k}_A}^{(E)} = 0, \\
\vec{\partial} \times \vec{A}_{\vec{k}_A}^{(E)} &= (-\partial_0 + iM)\vec{A}_{\vec{k}_A}^{(B)}, \quad \vec{\partial} \cdot \vec{A}_{\vec{k}_A}^{(B)} = 0
\end{aligned} \tag{20}$$

as a sufficient condition for the relativistic free one-particle wave equations\*

$$(\square - M^2)\vec{A}_{\vec{k}_A}^{(E,B)} = 0. \quad (21)$$

With Eqs. (12), the asymptotic soft relations (20) show that

$$\vec{k}_A \times \vec{e}^{(E)} = (\omega_A + M)\vec{e}^{(B)}, \quad \vec{k}_A \cdot \vec{e}^{(E)} = 0 \quad (22)$$

and  $\vec{k}_A \times \vec{e}^{(B)} = (-\omega_A + M)\vec{e}^{(E)}$ ,  $\vec{k}_A \cdot \vec{e}^{(B)} = 0$ , what gives jointly  $(-\vec{k}_A^2 + \omega_A^2 - M^2)\vec{e}^{(E,B)} = 0$ .<sup>†</sup>

Hence,

$$\vec{e}^{(B)} = \sqrt{\frac{\omega_A - M}{\omega_A + M}} \frac{\vec{k}_A}{|\vec{k}_A|} \times \vec{e}^{(E)} \quad (23)$$

( $\omega_A = \sqrt{\vec{k}_A^2 + M^2}$ ), both in right and lefthanded frame of reference. Thus,  $0 < \vec{e}^{(B)2} = (\omega_A - M)/(\omega_A + M) < 1$ , if  $\vec{e}^{(E)2} = 1$  and  $\omega_A > M > 0$ . Note that  $\vec{e}^{(B)} \rightarrow 0$ , when  $|\vec{k}_A|/M \rightarrow 0$  (as *e.g.* for an  $A$  boson at rest). On the contrary,  $\vec{e}^{(B)} \rightarrow (\vec{k}_A/|\vec{k}_A|) \times \vec{e}^{(E)}$ , when  $M/\omega_A \rightarrow 0$  (*i.e.*,  $M/|\vec{k}_A| \rightarrow 0$ ).

Concluding our presentation of the Maxwell-type new option for massive  $A$  bosons, we can say that now in their free wave functions there appear two not normalized to 1 axial vectors  $\vec{e}^{(B)}$ , dependent through the relations (23) on two (independent) polar vectors  $\vec{e}^{(E)}$  forming together with  $\vec{k}_A/|\vec{k}_A|$  a triple of orthonormal versors. So,

$$\begin{aligned} \vec{e}_a^{(E)} \cdot \vec{e}_b^{(E)} &= \delta_{ab} \quad (a, b = 1, 2) \quad , \quad \vec{e}_a^{(E)} \cdot \frac{\vec{k}_A}{|\vec{k}_A|} = 0 \quad (a, b = 1, 2) , \\ \sum_{a=1}^2 e_{ak}^{(E)} e_{al}^{(E)} + \frac{k_{Ak} k_{Al}}{\vec{k}_A^2} &= \delta_{kl} \quad (k, l = 1, 2, 3) \end{aligned} \quad (24)$$

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\*An equivalent compact form of the asymptotic soft constraint (20) is

$$(\partial_\nu + iMg_{0\nu})A_{\vec{k}_A}^{\mu\nu} = 0 \quad , \quad (\partial_\nu - iMg_{0\nu})\tilde{A}_{\vec{k}_A}^{\mu\nu} = 0,$$

where  $\tilde{A}^{\mu\nu} \equiv (1/2)\varepsilon^{\mu\nu\rho\sigma}A_{\rho\sigma}$  ( $\varepsilon^{0123} = 1$ ) and so  $A^{\mu\nu} \rightarrow \tilde{A}^{\mu\nu}$  when  $\vec{A}^{(E)} \rightarrow \vec{A}^{(B)}$  and  $\vec{A}^{(B)} \rightarrow -\vec{A}^{(E)}$  with  $M \rightarrow -M$ . Here,  $g_{0\nu}$  plays the role of a spurion for Lorentz boosts, spoiling relativistic covariance when  $M \neq 0$ . It disappears in the relativistic wave equations (21) fulfilled necessarily, if the condition (20) is satisfied.

<sup>†</sup>In this case, the kinematics of  $A$  bosons is relativistic,  $k_A^2 = M^2$ , but the antisymmetric polarization tensors  $e_{\mu\nu}$  are not relativistically covariant, since they satisfy the constraint

$$(k_{A\nu} - Mg_{0\nu})e^{\mu\nu} = 0 \quad , \quad (k_{A\nu} + Mg_{0\nu})\tilde{e}^{\mu\nu} = 0$$

involving the spurion  $g_{0\nu}$  when  $M \neq 0$  (here,  $\tilde{e}^{\mu\nu} = (1/2)\varepsilon^{\mu\nu\rho\sigma}e_{\rho\sigma}$ , thus  $e^{\mu\nu} \rightarrow \tilde{e}^{\mu\nu}$  when  $\vec{e}^{(E)} \rightarrow \vec{e}^{(B)}$  and  $\vec{e}^{(B)} \rightarrow -\vec{e}^{(E)}$  with  $M \rightarrow -M$ ). This compact form of constraint is equivalent to Eqs. (22) and two subsequent relations in the text. Note that the covariance of  $e_{\mu\nu}$  appears in the limit of  $M/\omega_A \rightarrow 0$ .

with  $\vec{e}_a^{(E)} = \left( e_{ak}^{(E)} \right)$  ( $a = 1, 2$ ,  $k = 1, 2, 3$ ). Then, two axial vectors  $\vec{e}_{1,2}^{(B)}$  are parallel and antiparallel to the polar vectors  $\vec{e}_{2,1}^{(E)}$ , respectively, if  $\vec{e}_1^{(E)} \times \vec{e}_2^{(E)} = \vec{k}_A/|\vec{k}_A|$  holds in the righthanded frame of reference. In such a new option, the coupling (9) *preserves the parity* in hidden sector, since  $\vec{e}^{(B)}$  are practically realized as axial vectors, while  $\vec{e}^{(E)}$  are polar vectors from the very beginning.

In this option, however, the asymptotic soft constraint (20) imposed on the free wave functions (12) of massive  $A$  bosons is not relativistically covariant as far as the antisymmetric polarization tensors are concerned, although the kinematics is relativistic. Thus, the Maxwell-type new option turns out to be not satisfactory for massive  $A$  bosons (when the full relativity does belong to our paradigm).

#### 4. Option of parallel $\vec{e}^{(E)}$ and $\vec{e}^{(B)}$ for $A$ bosons

In contrast to the relations (23) between  $\vec{e}_a^{(E)}$  and  $\vec{e}_a^{(B)}$  ( $a = 1, 2$ ), other possible relations between them (now with  $a = 1, 2, 3$ ), namely

$$\vec{e}_{1,2,3}^{(B)} = \vec{e}_{2,3,1}^{(E)} \times \vec{e}_{3,1,2}^{(E)} = \left[ \left( \vec{e}_1^{(E)} \times \vec{e}_2^{(E)} \right) \cdot \vec{e}_3^{(E)} \right] \vec{e}_{1,2,3}^{(E)}, \quad (25)$$

are relativistically covariant. In fact, using Eqs. (25), we obtain for antisymmetric polarization tensors  $e_{\mu\nu a}$  ( $a = 1, 2, 3$ ) the following forms

$$e_{\mu\nu a} e_a^{\mu\nu} = 2(\vec{e}_a^{(B)2} - \vec{e}_a^{(E)2}) = 2 \left\{ \left[ \left( \vec{e}_1^{(E)} \times \vec{e}_2^{(E)} \right) \cdot \vec{e}_3^{(E)} \right]^2 - 1 \right\} \vec{e}_a^{(E)2} = 2 [(\pm 1)^2 - 1] = 0 \quad (26)$$

( $a = 1, 2, 3$ ), being relativistically covariant in a trivial way, while in the case of relations (23) we get the forms

$$e_{\mu\nu a} e_a^{\mu\nu} = 2(\vec{e}_a^{(B)2} - \vec{e}_a^{(E)2}) = 2 \left( \frac{\omega_A - M}{\omega_A + M} - 1 \right) \vec{e}_a^{(E)2} = -\frac{4M}{\omega_A + M} \quad (27)$$

( $a = 1, 2$ ), violating the relativistic covariance when  $M \neq 0$ . When the relations (25) hold, Eqs. (13) are valid for  $\vec{e}_a^{(E,B)}$  as previously for independent  $\vec{e}_a^{(E)}$  and  $\vec{e}_a^{(B)}$  ( $a = 1, 2, 3$ ).

Note that in the case of relations (25) the corresponding axial and polar vectors  $\vec{e}_a^{(B)}$  and  $\vec{e}_a^{(E)}$  ( $a = 1, 2, 3$ ), are parallel or antiparallel in the right or lefthanded frame of reference, respectively (in these frames,  $\vec{e}_{2,3,1}^{(E)} \times \vec{e}_{3,1,2}^{(E)} = \pm \vec{e}_{1,2,3}^{(E)}$ ). An  $A$  boson displays



three independent polarizations that can be described by polar  $\vec{e}_a^{(E)}$ , since axial  $\vec{e}_a^{(B)}$  are practically realized by means of relations (25) in terms of polar  $\vec{e}_a^{(E)}$ .

Thus, in conclusion, the new option accepting the relations (25) between  $\vec{e}_a^{(E)}$  and  $\vec{e}_a^{(B)}$  ( $a = 1, 2, 3$ ) may be satisfactory in describing polarization of  $A$  bosons. It is a scheme practically realizing axial  $\vec{e}_a^{(B)}$  in terms of polar  $\vec{e}_a^{(E)}$  and being relativistically covariant. In this option, *the parity is preserved* by the coupling (9) in hidden sector.

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